Learning-based Max-Min Fair Hybrid Precoding for mmWave Multicasting

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Motivation

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- Multicast beamforming with fully-digital precoders has been widely studied in the literature.
- However, the benefits and challenges with hybrid precoders require additional study.
- We investigate the joint design of hybrid precoding and analog combining for max-min fairness single-group multicasting in millimeter-wave systems. We propose `LB-GDM`, a learning-based approach that leverages *(i)* gradient descent with momentum and *(ii)* alternating optimization.

Motivation

- Features of the proposed scheme `LB-GDM`
  - Has low complexity **[compared to SDR]**
  - Leverages alternating optimization **[several parameters]**
  - Is based on learning with gradient descent with momentum

- Our proposed design does not require:
  - Code-books
  - Solution with a fully-digital precoder.

Single-group Multicasting

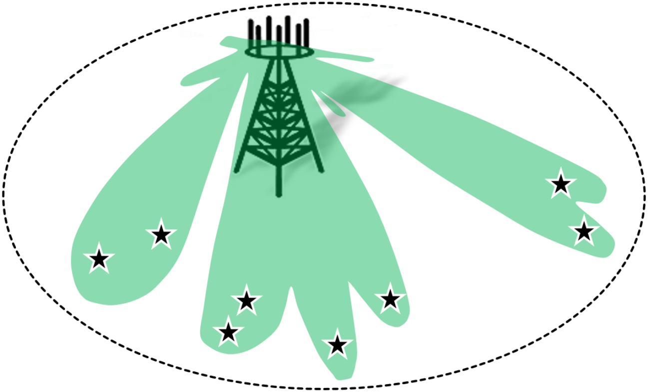
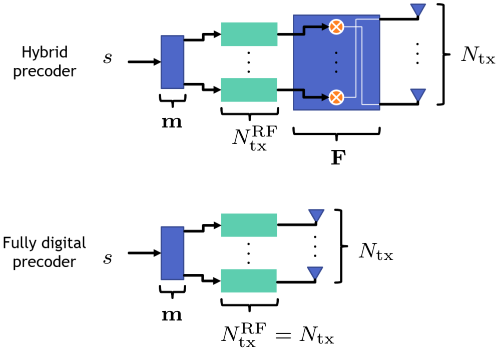


Figure: K-user Multicasting

Hybrid Precoder



$\mathbf{m} \in \mathbb{C}^{N_{\text{tx}}^{\text{RF}} \times 1}$: digital precoder

$\mathbf{F} \in \mathcal{F}^{N_{\text{tx}} \times N_{\text{tx}}^{\text{RF}}}$: analog precoder

$\mathcal{F} = \left\{ \sqrt{\delta_{\text{tx}}}, \ldots, \sqrt{\delta_{\text{tx}}} e^{\frac{2\pi (L_{\text{tx}} - 1)}{L_{\text{tx}}}} \right\}$: set of phase shifts

N_{tx}: number of transmit antennas

$N_{\text{tx}}^{\text{RF}}$: number of RF chains

L_{tx}: number of phase shifts

Figure: Hybrid and fully-digital precoders

System Model

The downlink signal is

$$\mathbf{x} = \mathbf{F}\mathbf{m}s \tag{1}$$

The received signal by user $k \in \mathcal{K}$ is

$$y_k = \underbrace{\mathbf{w}_k^H \mathbf{H}_k \mathbf{x}}_{\text{multicast signal}} + \underbrace{\mathbf{w}_k^H \mathbf{n}_k}_{\text{noise}}, \tag{2}$$

$\mathbf{w}_k$: combiner of the k-th user
$\mathbf{F}$: analog precoder
$\mathbf{m}$: digital precoder
$\mathbf{H}_k$: channel between the gNodeB and the k-th user
K: number of users
$\mathcal{K} = \{1, \ldots, K\}$: set of users
s: multicast symbol

System Model

The received signal by user $k \in \mathcal{K}$ is

$$y_k = \underbrace{\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m} s}_{\text{multicast signal}} + \underbrace{\mathbf{w}_k^H \mathbf{n}_k}_{\text{noise}}, \tag{3}$$

The SNR at user k is

$$\gamma_k = \frac{\left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m} \right|^2}{\sigma^2 \left\| \mathbf{w}_k \right\|_2^2}, \tag{4}$$

$\mathbf{w}_k$: combiner of the k-th user
$\mathbf{F}$: analog precoder
$\mathbf{m}$: digital precoder
$\mathbf{H}_k$: channel between the gNodeB and the k-th user
K: number of users
$\mathcal{K} = \{1, \ldots, K\}$: set of users
s: multicast symbol

Problem Formulation

$$\mathcal{P}_0^{\text{hyb}}: \max_{\mathbf{F},\mathbf{m},\{\mathbf{w}_k\}_{k=1}^{K}} \min_{k\in\mathcal{K}} \frac{\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}\right|^2}{\sigma^2 \left\|\mathbf{w}_k\right\|_2^2} \tag{5a}$$

$$\text{s.t.} \quad \left\|\mathbf{F}\mathbf{m}\right\|_2^2 = P_{\text{tx}}^{\max}, \tag{5b}$$

$$\left[\mathbf{F}\right]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}, \tag{5c}$$

$$\left\|\mathbf{w}_k\right\|_2^2 = P_{\text{rx}}^{\max}, k \in \mathcal{K}, \tag{5d}$$

$$\left[\mathbf{w}_k\right]_l \in \mathcal{W}, l \in \mathcal{L}, \forall k \in \mathcal{K}, \tag{5e}$$

$\mathcal{F} = \left\{\sqrt{\delta_{\text{tx}}}, \ldots, \sqrt{\delta_{\text{tx}}} e^{j\frac{2\pi(L_{\text{tx}}-1)}{L_{\text{tx}}}}\right\}$: allowed phase shifts at the precoder

$\mathcal{W} = \left\{\sqrt{\delta_{\text{rx}}}, \ldots, \sqrt{\delta_{\text{rx}}} e^{j\frac{2\pi(L_{\text{rx}}-1)}{L_{\text{rx}}}}\right\}$: allowed phase shifts at the combiners

L_{tx}: number of phase shifts at the precoder

L_{rx}: number of phase shifts at the combiners

Proposed Solution

$$\mathcal{P}_1^{\text{hyb}}: \max_{\mathbf{F}} \min_{k\in\mathcal{K}} \quad \frac{\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}\right|^2}{\sigma^2 P_{\text{rx}}^{\max}} \tag{6a}$$

$$\text{s.t.} \quad \|\mathbf{F}\mathbf{m}\|_2^2 = P_{\text{tx}}^{\max}, \tag{6b}$$

$$[\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}. \tag{6c}$$

$$\mathcal{P}_2^{\text{hyb}}: \max_{\mathbf{m}} \min_{k\in\mathcal{K}} \quad \left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}\right|^2 \tag{7a}$$

$$\text{s.t.} \quad \|\mathbf{F}\mathbf{m}\|_2^2 = P_{\text{tx}}^{\max}. \tag{7b}$$

$$\mathcal{P}_3^{\text{hyb}}: \max_{\{\mathbf{w}_k\}_{k=1}^K} \min_{k\in\mathcal{K}} \quad \frac{\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}\right|^2}{\sigma^2 \left\|\mathbf{w}_k\right\|_2^2} \tag{8a}$$

$$\text{s.t.} \quad [\mathbf{w}_k]_l \in \mathcal{W}, l \in \mathcal{L}, \forall k \in \mathcal{K}. \tag{8b}$$

Optimization of the Analog Precoder $\mathbf{F}$

$$\mathcal{P}_1^{\mathsf{hyb}} : \max_{\mathbf{F}} \ \min_{k \in \mathcal{K}} \quad \frac{\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}\right|^2}{\sigma^2 P_{\mathrm{rx}}^{\max}} \tag{9a}$$

$$\text{s.t.} \quad \|\mathbf{F}\mathbf{m}\|_2^2 = P_{\mathrm{tx}}^{\max}, \tag{9b}$$

$$[\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}. \tag{9c}$$

We equivalently recast $\mathcal{P}_1^{\mathsf{hyb}}$ as $\overline{\mathcal{P}}_1^{\mathsf{hyb}}$

$$\overline{\mathcal{P}}_1^{\mathsf{hyb}} : \max_{\mathbf{F}} \ \min_{k \in \mathcal{K}} \quad \frac{\mathbf{m}^H \mathbf{F}^H \mathbf{H}_k^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}}{\mathbf{m}^H \mathbf{F}^H \mathbf{F} \mathbf{m}} \tag{10a}$$

$$\text{s.t.} \quad [\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}. \tag{10b}$$

Optimization of the Analog Precoder $\mathbf{F}$

Instead of approaching (10), we propose to solve the surrogate problem (11), which consists of a weighted sum of all $\tau_k^F = \frac{\mathbf{m}^H\mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\mathbf{m}}{\mathbf{m}^H\mathbf{F}^H\mathbf{F}\mathbf{m}}$, as shown in (11)

$$\widehat{\mathcal{P}}_1^{\text{hyb}} : \max_{\mathbf{F}} \quad \sum_{k=1}^{K} c_k \frac{\mathbf{m}^H\mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\mathbf{m}}{\mathbf{m}^H\mathbf{F}^H\mathbf{F}\mathbf{m}} \tag{11a}$$

$$\text{s.t.} \quad [\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}, \tag{11b}$$

where $c_k \geq 0$ denotes the k-th weighting factor

Optimization of the Analog Precoder $\mathbf{F}$

Notice that

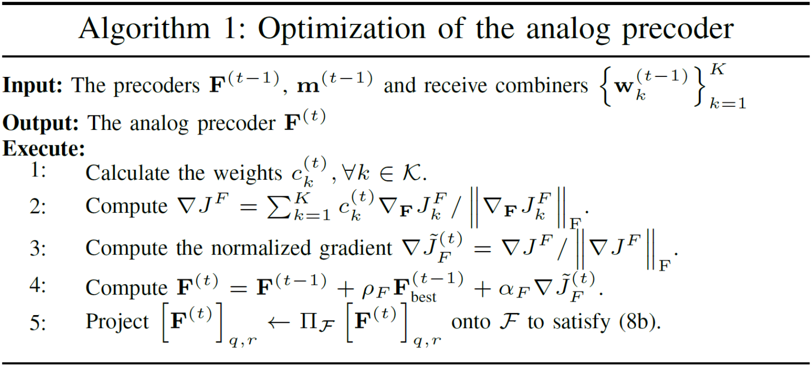
$$\begin{aligned}\tau_k^F &\leq \lambda_{\mathsf{max}}\left(\left(\mathbf{F}^H\mathbf{F}\right)^{-1}\mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\right)\\ &= \underbrace{\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\left(\mathbf{F}^H\mathbf{F}\right)^{-1}\mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k,}_{J_k^F}\end{aligned} \tag{12}$$

where $\lambda_{\mathsf{max}}(\cdot)$ extracts the maximum eigenvalue. Upon replacing τ_k^F in (11) by its upper bound J_k^F, the problem collapses to

$$\widetilde{\mathcal{P}}_1^{\mathsf{hyb}}: \max_{\mathbf{F}} \quad \sum_{k=1}^{K} c_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\left(\mathbf{F}^H\mathbf{F}\right)^{-1}\mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k, \tag{13a}$$

$$\text{s.t.} \quad [\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}. \tag{13b}$$

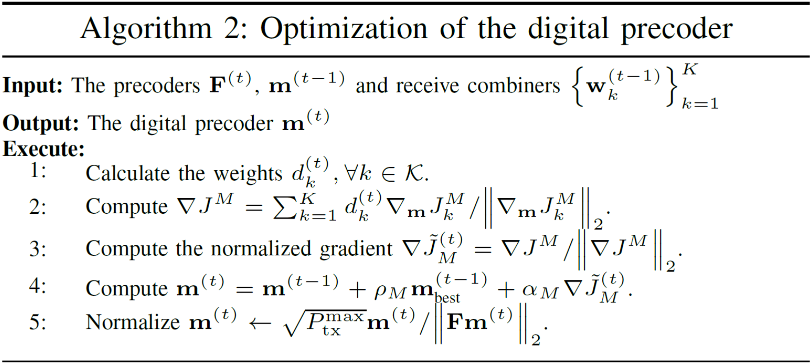
Optimization of the Analog Precoder $\mathbf{F}$



Algorithm 1: Optimization of the analog precoder

```
Input: The precoders $\mathbf{F}^{(t-1)}$, $\mathbf{m}^{(t-1)}$ and receive combiners $\left\{\mathbf{w}_k^{(t-1)}\right\}_{k=1}^{K}$
Output: The analog precoder $\mathbf{F}^{(t)}$
Execute:
    Calculate the weights $c_k^{(t)}, \forall k \in \mathcal{K}$.
    Compute $\nabla J^F = \sum_{k=1}^{K} c_k^{(t)} \nabla_{\mathbf{F}} J_k^F / \left\| \nabla_{\mathbf{F}} J_k^F \right\|_{\mathrm{F}}$.
    Compute the normalized gradient $\nabla \tilde{J}_F^{(t)} = \nabla J^F / \left\| \nabla J^F \right\|_{\mathrm{F}}$.
    Compute $\mathbf{F}^{(t)} = \mathbf{F}^{(t-1)} + \rho_F \mathbf{F}_{\text{best}}^{(t-1)} + \alpha_F \nabla \tilde{J}_F^{(t)}$.
    Project $\left[\mathbf{F}^{(t)}\right]_{q,r} \leftarrow \Pi_{\mathcal{F}} \left[\mathbf{F}^{(t)}\right]_{q,r}$ onto $\mathcal{F}$ to satisfy (8b).
```

Optimization of the Digital Precoder $\mathbf{m}$



Algorithm 2: Optimization of the digital precoder

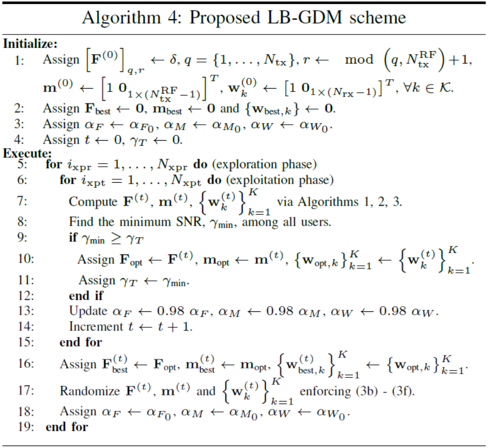
```
Input: The precoders $\mathbf{F}^{(t)}$, $\mathbf{m}^{(t-1)}$ and receive combiners $\left\{\mathbf{w}_k^{(t-1)}\right\}_{k=1}^{K}$
Output: The digital precoder $\mathbf{m}^{(t)}$
Execute:
    Calculate the weights $d_k^{(t)}, \forall k \in \mathcal{K}$.
    Compute $\nabla J^M = \sum_{k=1}^{K} d_k^{(t)} \nabla_{\mathbf{m}} J_k^M / \left\| \nabla_{\mathbf{m}} J_k^M \right\|_2$.
    Compute the normalized gradient $\nabla \tilde{J}_M^{(t)} = \nabla J^M / \left\| \nabla J^M \right\|_2$.
    Compute $\mathbf{m}^{(t)} = \mathbf{m}^{(t-1)} + \rho_M \mathbf{m}_{\text{best}}^{(t-1)} + \alpha_M \nabla \tilde{J}_M^{(t)}$.
    Normalize $\mathbf{m}^{(t)} \leftarrow \sqrt{P_{\text{tx}}^{\max}} \mathbf{m}^{(t)} / \left\| \mathbf{F}\mathbf{m}^{(t)} \right\|_2$.
```

Optimization of the Analog Combiner $\mathbf{w}_k$

Algorithm 3: Optimization of the k-th combiner

```
Input: The precoders $\mathbf{F}^{(t)}$, $\mathbf{m}^{(t)}$ and the receive combiner $\mathbf{w}_k^{(t-1)}$
Output: The receive combiner $\mathbf{w}_k^{(t)}$
Execute:
    Compute $\nabla_{\mathbf{w}_k} J_k^W$.
    Compute $\nabla_{\mathbf{w}_k} \tilde{J}_W^{(t)} = \nabla_{\mathbf{w}_k} J_k^W / \left\| \nabla_{\mathbf{w}_k} J_k^W \right\|_2$.
    Compute $\mathbf{w}_k^{(t)} = \mathbf{w}_k^{(t-1)} + \rho_W \mathbf{w}_{\text{best},k}^{(t-1)} + \alpha_W \nabla_{\mathbf{w}_k} \tilde{J}_W^{(t)}$.
    Project $\left[\mathbf{w}_k^{(t)}\right]_l \leftarrow \Pi_{\mathcal{W}} \left[\mathbf{w}_k^{(t)}\right]_l$ onto $\mathcal{W}$, $\forall l \in \mathcal{L}$ to satisfy (12b).
```

Optimization Algorithm



Algorithm 4: Proposed LB-GDM scheme

```
Initialize:
  Assign $\left[\mathbf{F}^{(0)}\right]_{q,r} \leftarrow \delta$, $q = \{1, \ldots, N_{\text{tx}}\}$, $r \leftarrow \mod\left(q, N_{\text{tx}}^{\text{RF}}\right) + 1$,
    $\mathbf{m}^{(0)} \leftarrow \left[1\ \mathbf{0}_{1\times(N_{\text{tx}}^{\text{RF}}-1)}\right]^T$, $\mathbf{w}_k^{(0)} \leftarrow [1\ \mathbf{0}_{1\times(N_{\text{rx}}-1)}]^T$, $\forall k \in \mathcal{K}$.
  Assign $\mathbf{F}_{\text{best}} \leftarrow \mathbf{0}$, $\mathbf{m}_{\text{best}} \leftarrow \mathbf{0}$ and $\{\mathbf{w}_{\text{best},k}\} \leftarrow \mathbf{0}$.
  Assign $\alpha_F \leftarrow \alpha_{F_0}$, $\alpha_M \leftarrow \alpha_{M_0}$, $\alpha_W \leftarrow \alpha_{W_0}$.
  Assign $t \leftarrow 0$, $\gamma_T \leftarrow 0$.
Execute:
  for $i_{\text{xpr}} = 1, \ldots, N_{\text{xpr}}$ do (exploration phase)
    for $i_{\text{xpt}} = 1, \ldots, N_{\text{xpt}}$ do (exploitation phase)
      Compute $\mathbf{F}^{(t)}$, $\mathbf{m}^{(t)}$, $\left\{\mathbf{w}_k^{(t)}\right\}_{k=1}^{K}$ via Algorithms 1, 2, 3.
      Find the minimum SNR, $\gamma_{\text{min}}$, among all users.
      if $\gamma_{\text{min}} \geq \gamma_T$
        Assign $\mathbf{F}_{\text{opt}} \leftarrow \mathbf{F}^{(t)}$, $\mathbf{m}_{\text{opt}} \leftarrow \mathbf{m}^{(t)}$, $\{\mathbf{w}_{\text{opt},k}\}_{k=1}^{K} \leftarrow \left\{\mathbf{w}_k^{(t)}\right\}_{k=1}^{K}$.
        Assign $\gamma_T \leftarrow \gamma_{\text{min}}$.
      end if
      Update $\alpha_F \leftarrow 0.98\ \alpha_F$, $\alpha_M \leftarrow 0.98\ \alpha_M$, $\alpha_W \leftarrow 0.98\ \alpha_W$.
      Increment $t \leftarrow t + 1$.
    end for
    Assign $\mathbf{F}_{\text{best}}^{(t)} \leftarrow \mathbf{F}_{\text{opt}}$, $\mathbf{m}_{\text{best}}^{(t)} \leftarrow \mathbf{m}_{\text{opt}}$, $\left\{\mathbf{w}_{\text{best},k}^{(t)}\right\}_{k=1}^{K} \leftarrow \{\mathbf{w}_{\text{opt},k}\}_{k=1}^{K}$.
    Randomize $\mathbf{F}^{(t)}$, $\mathbf{m}^{(t)}$ and $\left\{\mathbf{w}_k^{(t)}\right\}_{k=1}^{K}$ enforcing (3b) - (3f).
    Assign $\alpha_F \leftarrow \alpha_{F_0}$, $\alpha_M \leftarrow \alpha_{M_0}$, $\alpha_W \leftarrow \alpha_{W_0}$.
  end for
```

Simulation Results - Scenario I

Goal: Evaluate the impact of exploration (N_{xpr}) and exploitation (N_{xpt})

Table: Simulation parameters

Description	Symbol	Value	Units
Transmit power	$P_{\mathrm{tx}}^{\max}$	30	dBm
Receive power	$P_{\mathrm{rx}}^{\max}$	10	dBm
Noise power	σ^2	30	dBm
Number of users	K	30	-
Number of transmit antennas	N_{tx}	15	-
Number of receive antennas	N_{rx}	2	-
Number of RF chains (at the hybrid precoder)	$N_{\mathrm{tx}}^{\mathrm{RF}}$	6	-
Number of phase shifts at the precoder	L_{tx}	8	-
Number of phase shifts at the combiner	L_{rx}	4	-
Number of exploration instances	N_{xpr}	100	-
Number of exploitation instances	L_{xpt}	100	-
Momentum factor	$\rho_F = \rho_M = \rho_W$	0.90	-
Diminishing learning factor	$\alpha_F = \alpha_M = \alpha_W$	0.98	-

Simulation Results - Scenario I

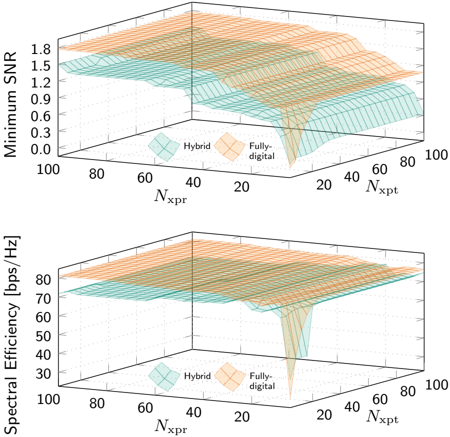


Figure: Impact of exploration ($N_{\rm xpr}$) and exploitation ($N_{\rm xpt}$).

Simulation Results - Scenario II

Goal: Evaluate the impact of the number of antennas N_{tx} and N_{rx}

Table: Simulation parameters

Description	Symbol	Value	Units
Transmit power	$P_{\mathrm{tx}}^{\max}$	30	dBm
Receive power	$P_{\mathrm{rx}}^{\max}$	10	dBm
Noise power	σ^2	30	dBm
Number of users	K	50	-
Number of transmit antennas	N_{tx}	$\{8, 12, 16\}$	-
Number of receive antennas	N_{rx}	$\{1, 2, 3, 4, 5\}$	-
Number of RF chains (at the hybrid precoder)	$N_{\mathrm{tx}}^{\mathrm{RF}}$	2	-
Number of phase shifts at the precoder	L_{tx}	8	-
Number of phase shifts at the combiner	L_{rx}	4	-
Number of exploration instances	N_{xpr}	100	-
Number of exploitation instances	L_{xpt}	100	-
Momentum factor	$\rho_F = \rho_M = \rho_W$	0.90	-
Diminishing learning factor	$\alpha_F = \alpha_M = \alpha_W$	0.98	-

Simulation Results - Scenario II

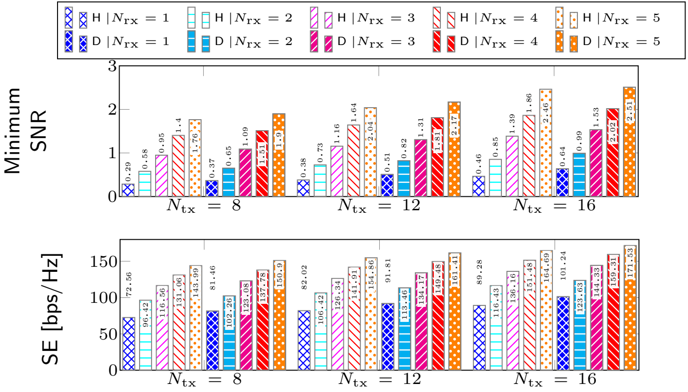


Figure: Performance evaluation of `LB-GDM` for varying N_{tx} and N_{rx} in fully-digital (D) and hybrid (H) precoders.

Simulation Results - Scenario III

Goal: Compare the performance with a SDR-based scheme

Table: Simulation parameters

Description	Symbol	Value	Units
Transmit power	$P_{\mathrm{tx}}^{\mathrm{max}}$	30	dBm
Receive power	$P_{\mathrm{rx}}^{\mathrm{max}}$	10	dBm
Noise power	σ^2	30	dBm
Number of users	K	$\{25, 50, 75, 100\}$	-
Number of transmit antennas	N_{tx}	20	-
Number of receive antennas	N_{rx}	3	-
Number of RF chains (at the hybrid precoder)	$N_{\mathrm{tx}}^{\mathrm{RF}}$	6	-
Number of phase shifts at the precoder	L_{tx}	8	-
Number of phase shifts at the combiner	L_{rx}	4	-
Number of exploration instances	N_{xpr}	120	-
Number of exploitation instances	L_{xpt}	120	-
Momentum factor	$\rho_F = \rho_M = \rho_W$	0.90	-
Diminishing learning factor	$\alpha_F = \alpha_M = \alpha_W$	0.98	-
Number of randomizations (`SDR-C`)	N_{rand}	$\{1, 10, 50, 100, 500, 1000\}$	-

Simulation Results - Scenario III

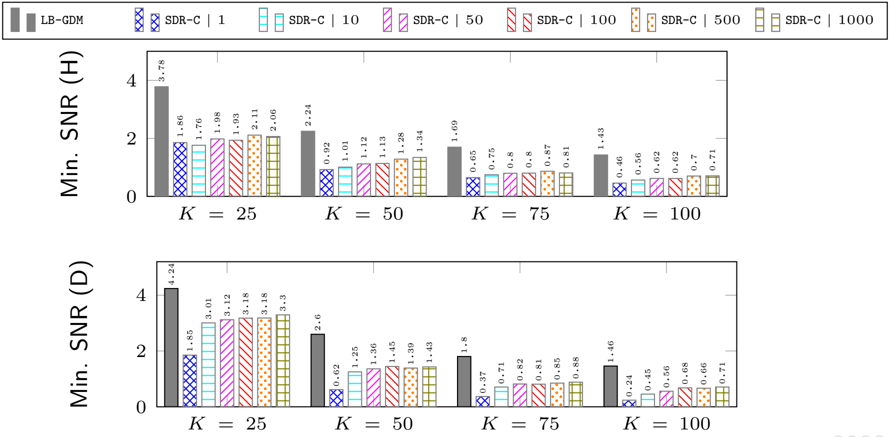


Figure: Performance comparison between `LB-GDM` and `SDR-C`

Conclusions

- We investigated the design of fully-digital and hybrid precoders for single-group multicasting using a learning-based scheme, `LB-GDM`.
- Our proposed low-complexity `LB-GDM` uses only matrix multiplications / additions and low-dimensional matrix inversion operations.
- We compare the performance of precoders based on `SDR-C` and `LB-GDM`. The results show that `LB-GDM` attains substantial additional gain for both digital and hybrid precoders.
- We corroborate the importance of incorporating more receive antennas. We achieve 75.7% and 100% gains in terms of the minimum SNR by increasing the number of receive antennas from one to two.

Questions

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