Hybrid Precoding for Multi-Group Multicasting in mmWave Systems

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Overview

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- Digital precoding for multicasting is a well-studied topic in the literature.
- However, its benefits and challenges for hybrid precoders in mmWave systems require additional study.
- We investigate the joint design of hybrid transmit precoders (with an arbitrary number of finite-resolution phase shifts) and receive combiners for mmWave multi-group multicasting.

Overview

- Our proposed is based on:
  - Semidefinite relaxation (SDR) **[convexification]**
  - Alternating optimization **[several parameters]**
  - Cholesky matrix factorization **[arbitrary phase shifts]**

- Our proposed design does not require:
  - Code-books
  - The optimal solution obtained by solving the problem with a fully-digital precoder.

Multi-group Multicasting

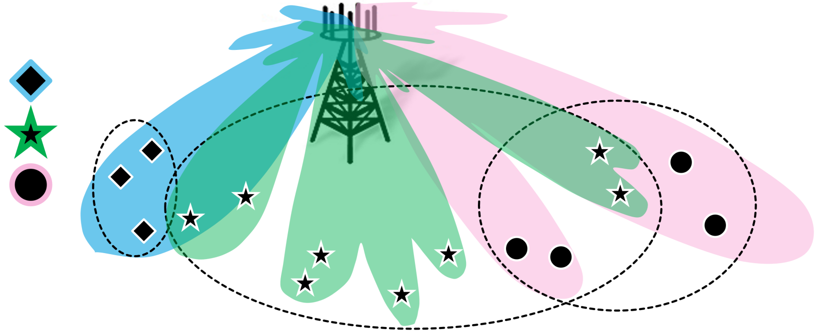

Figure: Multi-group Multicasting

Hybrid Precoder

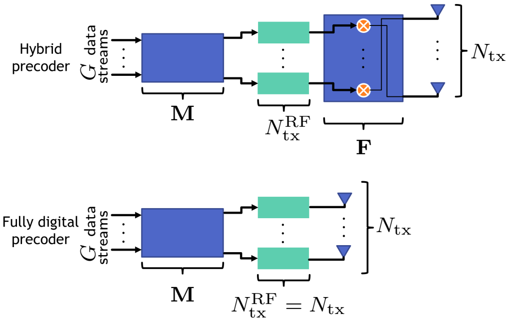

Figure: Multi-group Multicasting

$\mathbf{M} \in \mathbb{C}^{N_{\text{tx}}^{\text{RF}} \times G}$: digital precoder

$\mathbf{F} \in \mathcal{F}^{N_{\text{tx}} \times N_{\text{tx}}^{\text{RF}}}$: analog precoder

$\mathcal{F} = \left\{ \sqrt{\delta}, \ldots, \sqrt{\delta} e^{\frac{2\pi(L-1)}{L}} \right\}$: set of phase shifts

N_{tx}: number of transmit antennas

$N_{\text{tx}}^{\text{RF}} \geq G$: number of RF chains

L: number of phase shifts

G: number of multicast groups

System Model

Downlink signal

$$\mathbf{x} = \mathbf{F}\mathbf{M}\mathbf{s} = \mathbf{F}\left[\mathbf{m}_1, \ldots, \mathbf{m}_G\right]\left[s_1, \ldots, s_G\right]^T \quad (1)$$

Received signal by user $k \in \mathcal{G}_i$, $i \in \mathcal{I}$

$$y_k = \mathbf{w}_k^H \left(\mathbf{H}_k \mathbf{x} + \mathbf{n}_k\right)$$

$$y_k = \underbrace{\mathbf{w}_k^H \mathbf{H}_k \sum_{j=1}^{G} \mathbf{F}\mathbf{m}_j s_j}_{\text{aggregate multicast signals}} + \underbrace{\mathbf{w}_k^H \mathbf{n}_k}_{\text{noise}}, \quad (2)$$

$\mathcal{K} = \{1, 2, \ldots, K\}$: set of users
$\mathcal{I} = \{1, 2, \ldots, G\}$: set of groups
$\mathcal{G}_i$: set of user indices (in multicast group i)
s_i: symbol for multicast group i

System Model

$$y_k = \underbrace{\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}_i s_i}_{\text{desired multicast signal}} + \underbrace{\mathbf{w}_k^H \mathbf{H}_k \sum_{\substack{j=1 \\ j \neq i}}^{G} \mathbf{F} \mathbf{m}_j s_j}_{\text{interference}} + \underbrace{\mathbf{w}_k^H \mathbf{n}_k}_{\text{noise}}, \quad (3)$$

$$\mathrm{SINR}_k = \frac{\left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}_i \right|^2}{\sum_{j \neq i} \left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{m}_j \right|^2 + \sigma^2 \left\| \mathbf{w}_k \right\|_2^2}, k \in \mathcal{G}_i, \quad (4)$$

$\mathbf{w}_k$: combiner of the k-th user
$\mathbf{H}_k$: channel between the gNodeB and the k-th user
G: number of multicast groups
K: number of users
$\mathcal{K} = \{1, 2, \ldots, K\}$: set of users
$\mathcal{I} = \{1, 2, \ldots, G\}$: set of groups
$\mathcal{G}_i$: set of user indices (in multicast group i)
s_i: symbol for multicast group i

Problem Formulation

$$\mathcal{P}_0^{\text{hyb}}: \min_{\substack{\mathbf{F},\{\mathbf{m}_i\}_{i=1}^{G},\\ \{\mathbf{w}_k\}_{k=1}^{K},\{x_k\}_{k=1}^{K}}} \sum_{i=1}^{G} \|\mathbf{F}\mathbf{m}_i\|_2^2 + \beta \sum_{k=1}^{K} x_k \quad (5a)$$

$$\text{s.t.} \quad \frac{\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F}\mathbf{m}_i\right|^2 + x_k}{\sum_{j\neq i}\left|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F}\mathbf{m}_j\right|^2 + \sigma^2 \|\mathbf{w}_k\|_2^2} \geq \gamma_i, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \quad (5b)$$

$$\|\mathbf{w}_k\|_2^2 = P_{\text{rx}}^{\max}, k \in \mathcal{K}, \quad (5c)$$

$$[\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}, \quad (5d)$$

$$x_k \geq 0, \quad (5e)$$

$q \in \mathcal{Q} = \{1, 2, \ldots, N_{\text{tx}}\}$, $r \in \mathcal{R} = \{1, 2, \ldots, N_{\text{tx}}^{\text{RF}}\}$

Optimization of $\mathbf{F}$

$$\mathcal{P}_1^{\text{hyb}} : \min_{\mathbf{F}, \{x_k\}_{k=1}^K} \quad \sum_{i=1}^{G} \|\mathbf{F}\mathbf{m}_i\|_2^2 + \beta \sum_{k=1}^{K} x_k \tag{6a}$$

$$\text{s.t.} \quad \gamma_i \left(\sum_{j \neq i} \left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{F}\mathbf{m}_j \right|^2 + \sigma^2 \|\mathbf{w}_k\|_2^2 \right) - \left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{F}\mathbf{m}_i \right|^2 \leq x_k, \forall k \in \mathcal{G}_i, \forall i \in \mathcal{I}, \tag{6b}$$

$$[\mathbf{F}]_{q,r} \in \mathcal{F}, q \in \mathcal{Q}, r \in \mathcal{R}, \tag{6c}$$

$$x_k \geq 0, \tag{6d}$$

Optimization of $\mathbf{F}$: Change of variables

$$\mathcal{P}_1^{\text{hyb}}: \min_{\mathbf{f},\{x_k\}_{k=1}^{K}} \quad \sum_{i=1}^{G} \|\mathbf{J}_i \mathbf{f}\|_2^2 + \beta \sum_{k=1}^{K} x_k \tag{7a}$$

$$\text{s.t.} \quad \gamma_i \left(\sum_{j \neq i} \left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{J}_j \mathbf{f} \right|^2 + \sigma^2 \|\mathbf{w}_k\|_2^2 \right) - \left| \mathbf{w}_k^H \mathbf{H}_k \mathbf{J}_i \mathbf{f} \right|^2 \leq x_k, \forall k \in \mathcal{G}_i, i \in \mathcal{I}, \tag{7b}$$

$$[\mathbf{f}]_n \in \mathcal{F}, n \in \mathcal{N}, \tag{7c}$$

$$x \geq 0, \tag{7d}$$

where $\mathbf{F}\mathbf{m}_i = \mathbf{J}_i \mathbf{f}$, $\mathbf{J}_i = \mathbf{m}_i^T \otimes \mathbf{I}$, $\mathbf{f} = \text{vec}(\mathbf{F})$ and $\mathcal{N} = \left\{1, 2, \ldots, N_{\text{tx}}^{\text{RF}} N_{\text{tx}}\right\}$.

Optimization of $\mathbf{F}$: SDP Representation

$$\mathcal{P}^{\text{hyb}}_{\text{SDP},1}: \min_{\mathbf{D},\{x_k\}_{k=1}^{K}} \quad \sum_{i=1}^{G} \operatorname{Tr}\left(\mathbf{D}\mathbf{R}_i\right) + \beta \sum_{k=1}^{K} x_k \tag{8a}$$

$$\text{s.t.} \quad \operatorname{Tr}\left(\mathbf{D}\left(\gamma_i \sum_{j \neq i} \mathbf{V}_{j,k} - \mathbf{V}_{i,k}\right)\right) + \sigma^2 \gamma_i \left\|\mathbf{w}_k\right\|_2^2 \leq x_k, \forall k \in \mathcal{G}_i, i \in \mathcal{I}, \tag{8b}$$

$$[\mathbf{D}]_{n,n} = \delta, n \in \mathcal{N}, \tag{8c}$$

$$\operatorname{rank}\left(\mathbf{D}\right) = 1, \tag{8d}$$

$$\mathbf{D} \succcurlyeq \mathbf{0}, \tag{8e}$$

$$x_k \geq 0, \tag{8f}$$

where $\mathbf{D} = \mathbf{f}\mathbf{f}^H$, $\left\|\mathbf{J}_i\mathbf{f}\right\|_2^2 = \operatorname{Tr}\left(\mathbf{R}_i\mathbf{D}\right)$, $\mathbf{R}_i = \mathbf{J}_i^H\mathbf{J}_i$, $\left|\mathbf{w}_k^H\mathbf{H}_k\mathbf{J}_i\mathbf{f}\right|^2 = \operatorname{Tr}\left(\mathbf{V}_{i,k}\mathbf{D}\right)$ and $\mathbf{V}_{i,k} = \mathbf{J}_i^H\mathbf{H}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{J}_i$.

Optimization of $\mathbf{F}$: SDR Representation

$$\mathcal{P}_{\mathrm{SDR},1}^{\mathrm{hyb}}: \min_{\mathbf{D},\{x_k\}_{k=1}^{K}} \sum_{i=1}^{G} \mathrm{Tr}\left(\mathbf{D}\mathbf{R}_i\right) + \beta \sum_{k=1}^{K} x_k \tag{9a}$$

$$\text{s.t.} \quad \mathrm{Tr}\left(\mathbf{D}\left(\gamma_i \sum_{j \neq i} \mathbf{V}_{j,k} - \mathbf{V}_{i,k}\right)\right) + \sigma^2 \gamma_i \left\|\mathbf{w}_k\right\|_2^2 \leq x_k, \forall k \in \mathcal{G}_i, i \in \mathcal{I}, \tag{9b}$$

$$\left[\mathbf{D}\right]_{n,n} = \delta, n \in \mathcal{N}, \tag{9c}$$

$$\mathbf{D} \succcurlyeq \mathbf{0}, \tag{9d}$$

$$x_k \geq 0, \tag{9e}$$

Optimization of $\mathbf{F}$: Phase Recovery - Stage A_1

Stage A_1:

- Any element (n_1, n_2) of matrix $\mathbf{D}$ can be represented as $[\mathbf{D}]_{n_1,n_2} = [\mathbf{f}]_{n_1} [\mathbf{f}]^*_{n_2}$, $n_1, n_2 \in \mathcal{N} = \left\{1, 2, \ldots, N_{\text{tx}}^{\text{RF}} N_{\text{tx}}\right\}$
- We define a vector $\mathbf{u} \in \mathbb{C}^{N_{\text{tx}}^{\text{RF}} N_{\text{tx}} \times 1}$ such that $\|\mathbf{u}\|_2^2 = \mathbf{u}^H \mathbf{u} = 1$.
- We can express $[\mathbf{D}]_{n_1,n_2}$ in terms of $\mathbf{u}$, i.e., $[\mathbf{D}]_{n_1,n_2} = \left([\mathbf{f}]_{n_1} \mathbf{u}^T\right) \left([\mathbf{f}]^*_{n_2} \mathbf{u}^*\right)$.
- We assume that $\mathbf{q}_n = [\mathbf{f}]_n \mathbf{u}$.
- Thus, $\mathbf{D}$ can be recast as $\mathbf{D} = \mathbf{Q}^T \mathbf{Q}^*$ with $\mathbf{Q} = \left[\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N_{\text{tx}}^{\text{RF}} N_{\text{tx}}}\right]$.

Optimization of $\mathbf{F}$: Phase Recovery - Stage A_2

Stage A_2:

- We denote with $\widehat{\mathbf{D}}$ the solution returned by $\mathcal{P}_{\text{SDR},1}^{\text{hyb}}$
- Via Cholesky matrix factorization we obtain $\widehat{\mathbf{D}} = \widehat{\mathbf{Q}}^T \widehat{\mathbf{Q}}^*$, where $\widehat{\mathbf{Q}} = \left[\widehat{\mathbf{q}}_1, \widehat{\mathbf{q}}_2, \ldots, \widehat{\mathbf{q}}_{N_{\text{tx}}^{\text{RF}} N_{\text{tx}}} \right]$.
- We have derived a relation that associates $\widehat{\mathbf{f}}$ with $\{\widehat{\mathbf{q}}_n\}_{n=1}^{N_{\text{tx}}^{\text{RF}} N_{\text{tx}}}$ (via $\widehat{\mathbf{q}}_n = \left[\widehat{\mathbf{f}} \right]_n \widehat{\mathbf{u}}$). However, both are unknown.
- The premise that all $\widehat{\mathbf{q}}_n$ can be obtained from the same $\widehat{\mathbf{u}}$ cannot be guaranteed.

Optimization of $\mathbf{F}$: Phase Recovery - Stage A_2

- Thus, we aim at finding an approximate $\widehat{\mathbf{f}}$ and $\widehat{\mathbf{u}}$, such that $\widehat{\mathbf{q}}_n \approx \left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{u}}$, and whose error in the 2-norm sense is minimum.

$$\mathcal{P}_{\mathrm{LS}}^{\mathrm{hyb}}: \quad \min_{\widehat{\mathbf{u}}, \left[\widehat{\mathbf{f}}\right]_n} \quad \sum_{n=1}^{N_{\mathrm{tx}}^{\mathrm{RF}} N_{\mathrm{tx}}} \left\| \widehat{\mathbf{q}}_n - \left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{u}} \right\|_2^2 \tag{10a}$$

$$\text{s.t.} \quad \|\widehat{\mathbf{u}}\|_2^2 = 1, \tag{10b}$$

$$\left[\widehat{\mathbf{f}}\right]_n \in \mathcal{F}, n \in \mathcal{N}. \tag{10c}$$

Optimization of $\mathbf{F}$: Phase Recovery - Stage A_3

Stage A_3:

- Minimizing simultaneously over both $\widehat{\mathbf{q}}_n$ and $\widehat{\mathbf{u}}$ is challenging. If we assume that $\widehat{\mathbf{u}}$ is known, then we are required to solve

$$\widetilde{\mathcal{P}}_{\mathrm{LS}}^{\mathrm{hyb}} : \quad \min_{\left[\widehat{\mathbf{f}}\right]_n} \sum_{n=1}^{N_{\mathrm{tx}}^{\mathrm{RF}} N_{\mathrm{tx}}} \left\| \widehat{\mathbf{q}}_n - \left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{u}} \right\|_2^2 \tag{11a}$$

$$\text{s.t.} \quad \left[\widehat{\mathbf{f}}\right]_n \in \mathcal{F}, n \in \mathcal{N} \tag{11b}$$

- By expanding (11a), we obtain
$$\left\| \widehat{\mathbf{q}}_n - \left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{u}} \right\|_2^2 = \widehat{\mathbf{q}}_n^H \widehat{\mathbf{q}}_n - 2\mathfrak{Re}\left(\left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{q}}_n^H \widehat{\mathbf{u}} \right) + \left| \left[\widehat{\mathbf{f}}\right]_n \right|^2 \widehat{\mathbf{u}}^H \widehat{\mathbf{u}}.$$

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Optimization of $\mathbf{F}$: Phase Recovery - Stage A_3

Stage A_3:

- Thus, (11) is equivalent to

$$\widetilde{\mathcal{P}}_{\mathrm{LS}}^{\mathrm{hyb}}: \quad \max_{\left[\widehat{\mathbf{f}}\right]_n} \sum_{n=1}^{N_{\mathrm{tx}}^{\mathrm{RF}} N_{\mathrm{tx}}} \mathfrak{Re}\left(\left[\widehat{\mathbf{f}}\right]_n \widehat{\mathbf{q}}_n^H \widehat{\mathbf{u}}\right) \tag{12a}$$

$$\text{s.t.} \quad \left[\widehat{\mathbf{f}}\right]_n \in \mathcal{F}, n \in \mathcal{N}. \tag{12b}$$

- Since $z_n = \widehat{\mathbf{q}}_n^H \widehat{\mathbf{u}}$ is known, (12a) is maximized when $\left[\widehat{\mathbf{f}}\right]_n \in \mathcal{F}$ is chosen with the closest phase to z_n^*.
- We solve $\widetilde{\mathcal{P}}_{\mathrm{LS}}^{\mathrm{hyb}}$ for N_{rand} candidate vectors $\widehat{\mathbf{u}}$ and select the choice that attains the minimum objective function value.

Optimization of $\mathbf{M}$

$$\mathcal{P}^{\text{hyb}}_{\text{SDR},2}: \min_{\substack{\{\mathbf{M}_i\}_{i=1}^{G},\\ \{x_k\}_{k=1}^{K}}} \quad \sum_{i=1}^{G} \operatorname{Tr}\left(\mathbf{Y}\mathbf{M}_i\right) + \beta \sum_{k=1}^{K} x_k \tag{13a}$$

$$\text{s.t.} \quad \operatorname{Tr}\left(\mathbf{X}_k\left(\gamma_i \sum_{j\neq i} \mathbf{M}_j - \mathbf{M}_i\right)\right) + \sigma^2\gamma_i \left\|\mathbf{w}_k\right\|_2^2 \leq x_k, \tag{13b}$$

$$\mathbf{M}_i \succcurlyeq \mathbf{0}, \tag{13c}$$

$$x_k \geq 0, \forall k \in \mathcal{G}_i, i \in \mathcal{I}, \tag{13d}$$

where $\mathbf{Y} = \mathbf{F}^H\mathbf{F}$, $\mathbf{X}_k = \mathbf{F}^H\mathbf{H}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}$ and $\mathbf{M}_i = \mathbf{m}_i\mathbf{m}_i^H$.

Optimization of $\{\mathbf{w}\}_{k=1}^{K}$

$$\mathcal{P}_{\text{SDR},3}^{\text{hyb}} : \min_{\substack{\{\mathbf{W}_k\}_{k=1}^{K}, \\ \{x_k\}_{k=1}^{K}}} \quad \sum_{k=1}^{K} x_k \tag{14a}$$

$$\text{s.t.} \quad \operatorname{Tr}\left(\mathbf{W}_k \left(\gamma_i \sum_{j \neq i} \mathbf{Z}_{k,j} - \mathbf{Z}_{k,i}\right)\right) + \sigma^2 \gamma_i \operatorname{Tr}\left(\mathbf{W}_k\right) \leq x_k, \tag{14b}$$

$$\operatorname{Tr}\left(\mathbf{W}_k\right) = P_{\text{rx}}^{\max}, \tag{14c}$$

$$\mathbf{W}_k \succcurlyeq \mathbf{0}, \tag{14d}$$

$$x_k \geq 0, \forall k \in \mathcal{G}_i, i \in \mathcal{I}, \tag{14e}$$

where $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ and $\mathbf{Z}_{k,i} = \mathbf{H}_k \mathbf{F} \mathbf{m}_i \mathbf{m}_i^H \mathbf{F}^H \mathbf{H}_k^H$.

Simulation Results - Scenario 1

Goal: Evaluate the performance of the hybrid and fully-digital precoders when $N_{\mathrm{tx}}^{\mathrm{RF}}$ and γ are varied

Table: Simulation parameters

Description	Symbol	Value	Units
Number of users	K	60	-
Number of groups	G	4	-
Receive power	-	10	dBm
Noise power	σ^2	10	dBm
Number of transmit antennas	N_{tx}	12	-
Number of receive antennas	N_{rx}	2	-
Number of randomization	N_{rand}	500	-
Number of iterations	N_{iter}	3	-
Number of simulations	-	100	-
SINR requirement	$\gamma_i = \gamma$	$\{4, 6, 8\}$	-
Number of RF chains	$N_{\mathrm{tx}}^{\mathrm{RF}}$	$\{5, 6, 7, 8, 9, 10, 11\}$	-

Simulation Results - Scenario 1

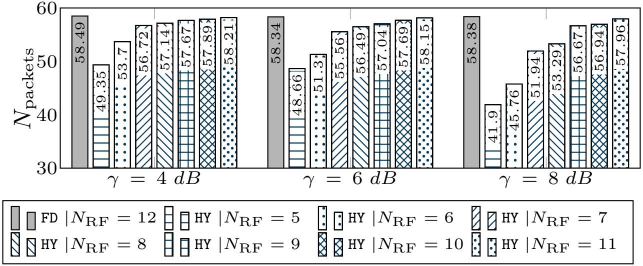

Figure: Evaluation of the number of decoded packets for $N_{\text{tx}} = 12$ when γ and $N_{\text{tx}}^{\text{RF}}$ are varied.

Simulation Results - Scenario 1

Goal: Evaluate the performance of the hybrid and fully-digital precoders when N_{rx} is varied

Table: Simulation parameters

Description	Symbol	Value	Units
Number of users	K	60	-
Number of groups	G	4	-
Receive power	-	10	dBm
Noise power	σ^2	10	dBm
Number of transmit antennas	N_{tx}	12	-
Number of receive antennas	N_{rx}	$\{2, 3, 4, 5\}$	-
Number of iterations	N_{iter}	4	-
Number of simulations	-	100	-
SINR requirement	$\gamma_i = \gamma$	$\{5\}$	-
Number of RF chains	$N_{\mathrm{tx}}^{\mathrm{RF}}$	8	-

Simulation Results - Scenario 2

Goal: Evaluate the performance of the hybrid and fully-digital precoders when N_{rx} is varied

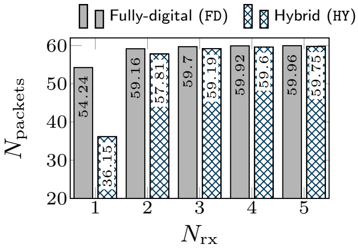

Figure: Evaluation of the number of decoded packets when N_{rx} is varied.

Simulation Results - Scenario 3

Goal: Evaluate the performance of the hybrid and fully-digital precoders when N_{rand} and N_{iter} are varied

Table: Simulation parameters

Description	Symbol	Value	Units
Number of users	K	60	-
Number of groups	G	4	-
Receive power	-	10	dBm
Noise power	σ^2	10	dBm
Number of transmit antennas	N_{tx}	12	-
Number of receive antennas	N_{rx}	2	-
Number of randomization	N_{rand}	$\{1, 10, 25, 50, 75, 100, 500, 1000\}$	-
Number of iterations	N_{iter}	$\{1, 2, 3, 4, \}$	-
Number of iterations	N_{iter}	4	-
Number of simulations	-	100	-
SINR requirement	$\gamma_i = \gamma$	5	-
Number of RF chains	$N_{\text{tx}}^{\text{RF}}$	8	-

Simulation Results - Scenario 2

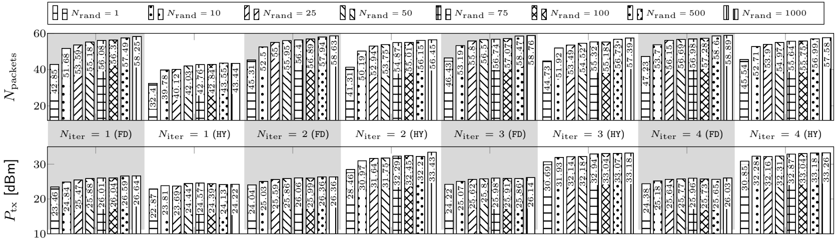

Figure: Evaluation of the number of decoded packets and transmit power for $N_{\text{tx}} = 12$ when N_{iter} and N_{rand} are varied.

Conclusions

- We investigated the optimization of multi-group multicast hybrid precoders in mmWave systems
- Our proposed solution is based on the alternating optimization, semidefinite relaxation and Cholesky decomposition.
- Our formulation allows the employment of an arbitrary number of phase shifts.
- We corroborated through simulations that the hybrid precoder can attain similar performance as its fully-digital counterpart.
- We show that having receivers with two antennas suffices to improve the number of decoded packets (up to 60% gain).

Questions

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Appendix

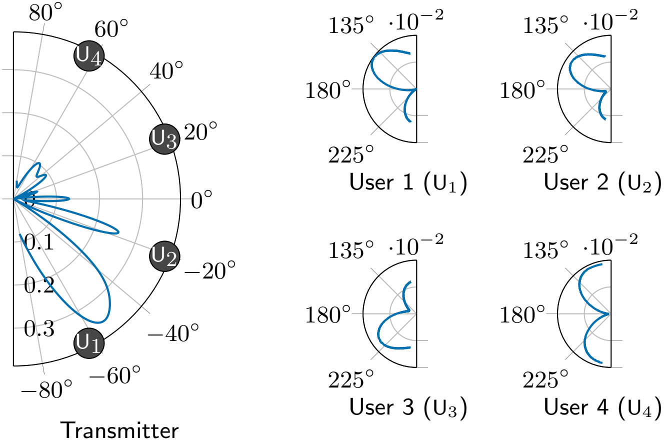

Figure: Radiation patterns

Appendix

Computational complexity: Neglecting the complexity owing to randomization and obviating the insignificant complexity increase due to the inclusion of slack parameters, the computational complexity of the proposed scheme when $N_{\text{iter}} = 1$ is

$$\mathcal{O}\left(\left(N_{\text{tx}}^{\text{RF}} N_{\text{tx}}\right)^6 + K\left(N_{\text{tx}}^{\text{RF}} N_{\text{tx}}\right)^2\right) + \mathcal{O}\left(G^3\left(N_{\text{tx}}^{\text{RF}}\right)^6 + KG\left(N_{\text{tx}}^{\text{RF}}\right)^2\right) + \mathcal{O}\left(K\left(N_{\text{rx}}\right)^6 + K\left(N_{\text{rx}}\right)^4\right).$$

Appendix

Algorithm 1: Proposed Iterative Approach

Define

Let $g^{(t)} = \sum_{i=1}^{G} \left\| \mathbf{F}^{(t)} \mathbf{m}_i^{(t)} \right\|_2^2$ be the total transmit power.

Let $\mathcal{K}^{(t)}$ be the number of users that satisfy (5b) at iteration t.

Initialize

Set $\mathbf{w}_k^{(0)} \leftarrow [1\ \mathbf{0}]^T, \forall k \in \mathcal{K}$, $\mathbf{m}_i^{(0)} \leftarrow [1\ \mathbf{0}]^T, \forall i \in \mathcal{I}$.

Set $\widetilde{\mathcal{K}} \leftarrow 0$, $\tilde{g} \leftarrow 10^5$, $t \leftarrow 1$.

Iterate

Set $C_1 \leftarrow 0$, $C_2 \leftarrow 0$ and $\{C_{3,k}\}_{k=1}^{K} \leftarrow 0$.

Optimize $\mathbf{F}$:

Solve $\mathcal{P}_{\mathrm{SDR},1}^{\mathrm{hyb}}$ to obtain $\mathbf{D}^{(t)}$.

repeat

Generate $\mathbf{u}$ with uniform distribution in the sphere $\|\mathbf{u}\|_2^2 = 1$.

Solve $\widetilde{\mathcal{P}}_{\mathrm{LS}}^{\mathrm{hyb}}$ and compute $\mathbf{F}^{(t)}$.

if $\mathcal{K}^{(t)} > \widetilde{\mathcal{K}}$ or $\left(\mathcal{K}^{(t)} = \widetilde{\mathcal{K}} \text{ and } g^{(t)} \leq \tilde{g}\right)$

Assign $\mathbf{F} \leftarrow \mathbf{F}^{(t)}$, $\tilde{g} \leftarrow g^{(t)}$, $\widetilde{\mathcal{K}} \leftarrow \mathcal{K}^{(t)}$.

end

Increase the counter C_1, $C_1 \leftarrow C_1 + 1$.

while $C_1 \leq N_{\mathrm{rand}}$

Optimize $\mathbf{m}_i$:

Solve $\mathcal{P}_{\mathrm{SDR},2}^{\mathrm{hyb}}$ and obtain $\left\{\mathbf{M}_i^{(t)}\right\}_{i=1}^{G}$.

repeat

Generate $\widetilde{\mathbf{m}}_i^{(t)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{M}_i^{(t)}\right), \forall i \in \mathcal{I}$.

if $\mathcal{K}^{(t)} > \widetilde{\mathcal{K}}$ or $\left(\mathcal{K}^{(t)} = \widetilde{\mathcal{K}} \text{ and } g^{(t)} \leq \tilde{g}\right)$

Assign $\{\mathbf{m}_i\}_{i=1}^{G} \leftarrow \left\{\mathbf{m}_i^{(t)}\right\}_{i=1}^{G}$, $\tilde{g} \leftarrow g^{(t)}$, $\widetilde{\mathcal{K}} \leftarrow \mathcal{K}^{(t)}$.

end

Increase the counter C_2, $C_2 \leftarrow C_2 + 1$.

while $C_2 \leq N_{\mathrm{rand}}$

Optimize $\mathbf{w}_k$:

Solve $\mathcal{P}_{\mathrm{SDR},3}^{\mathrm{hyb}}$ and obtain $\left\{\mathbf{W}_k^{(t)}\right\}_{k=1}^{K}$.

repeat for each k

Generate $\mathbf{w}_k^{(t)} \leftarrow \mathbf{W}_k^{(t)} \mathbf{v}_k, \forall k \in \mathcal{K}$ with $\mathbf{v}_k$ uniformly distributed in the sphere $\|\mathbf{v}_k\|_2^2 = 1$.

if $\mathcal{K}^{(t)} > \widetilde{\mathcal{K}}$ or $\left(\mathcal{K}^{(t)} = \widetilde{\mathcal{K}} \text{ and } g^{(t)} \leq \tilde{g}\right)$

Assign $\mathbf{w}_k \leftarrow \mathbf{w}_k^{(t)}$, $\tilde{g} \leftarrow g^{(t)}$, $\widetilde{\mathcal{K}} \leftarrow \mathcal{K}^{(t)}$

end

Increase the counter $C_{3,k}$, $C_{3,k} \leftarrow C_{3,k} + 1$

while $C_{3,k} \leq \lfloor N_{\mathrm{rand}}/K \rfloor$

Until $t > N_{\mathrm{iter}}$

Figure: Algorithm